

Emergent Universe in Brane World Scenario with Schwarzschild-de Sitter Bulk

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A model of an emergent universe is obtained in brane world. Here the bulk energy is in the form of cosmological constant, while the brane consists of a fluid satisfying an equation of state of the form $p_b = \frac{1}{3} \rho_b$, which is effectively a radiation equation of state at high energies. It is shown that with the positive bulk cosmological constant, one of our models represents an emergent universe.

The search for singularity free inflationary models in the context of Classical General Relativity has recently led to the development of emergent universes.

In 1967 Harrison [1] obtained a model of the closed universe containing radiation, which approaches the state of an Einstein static model asymptotically, i.e, as $t \rightarrow -\infty$. This kind of model has so far been discovered subsequently by several workers in the recent past such as that of Ellis and Maartens [2], Ellis et al. [3]. They obtained closed universes with a minimally coupled scalar field ϕ with a special form for self interacting potential and possibly some ordinary matter with equation of state $p = \omega \rho$ where $-\frac{1}{3} \leq \omega \leq 1$. However, exact analytic solutions were not presented in these models, although their behaviour alike that of an emergent universe was highlighted. An emergent universe is a model universe in which there is no timelike singularity, is ever existing and having almost static behaviour in the infinite past ($t \rightarrow -\infty$) as is mentioned earlier. The model eventually evolves into an inflationary stage. In fact, the emergent universe scenario can be said to be a modern version and extension of the original Lemaitre-Eddington universe. Mukherjee et al. [4] obtained solutions for Starobinsky model for flat FRW space time and studied the features of an emergent universe. Very recently, a general framework for an emergent universe model has been proposed by Mukherjee et al. [5] using an adhoc equation of state connecting the pressure and density. However, these solutions require exotic matter in many cases. Such models are appealing since they provide specific examples of non singular (geodesically complete) inflationary universes. Further, it has been proposed that entropy considerations favor the Einstein static state as the initial state for our universe [6].

The emergent universe models mentioned above for four dimensional space-time assume features like positive spatial curvature, minimally coupled scalar field or exotic matter. So far, we have not noticed any emergent universe model in the brane with the background of a bulk with cosmological constant. In the present work, we consider a perfect fluid satisfying an equation of state

$$p_b = \frac{1}{3} \rho_b \quad (1)$$

The evolution of the brane world models in which we live, in the special case, interestingly shows the feature of an emergent universe behaviour. The models described here are not only homogeneous and isotropic at large scale but also are spatially flat.

The brane energy density consists of two parts, one due to the ordinary energy density and the other due to the so called brane tension. The brane energy density and pressure are therefore given respectively as $\rho_b = \rho + \sigma$ and $p_b = p - \sigma$. Initially at large density $\rho \gg \sigma$ so that $p \approx \frac{1}{3} \rho$, which is effectively a radiation equation of state. On the other hand, as we show in what follows that at late stage, that is at $a_0 \rightarrow \infty$ and $\rho \rightarrow 0$, the brane inflates. There are not so many exact cosmological solutions in the brane because the energy density of the brane appears quadratically in the modified Friedmann equations instead of its linear behaviour as in the usual equations. We believe that particularly the emergent universe model in the brane of our present paper is the first of its kind in the existing literature.

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The geometry of the five dimensional bulk is assumed to be characterized by the space time metric of the form

$$ds^2 = -n^2(t, y)dt^2 + a^2(t, y)\delta_{ij}dx^i dx^j + b^2(t, y)dy^2 \quad (2)$$

where y is the fifth coordinate and the hypersurface $y = 0$ is identified as the world volume of the brane that forms our universe. For simplicity, we choose the usual spatial section of the brane to be flat. Now following Binétry et al. [7, 8], the energy conservation equation on the brane reads

$$\dot{\rho}_b + 3(\rho_b + p_b)\frac{\dot{a}_0}{a_0} = 0 \quad (3)$$

which integrating once (using the equation of state (1)) gives

$$\rho_b = \frac{\rho_0}{a_0^4} \quad (4)$$

(ρ_0 , an arbitrary integration constant)

Using this form for ρ_b , the generalized Friedmann type equations take the form (see eqns. (45) and (46) in ref. [9])

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^4 \rho_b^2}{36} + \frac{\kappa^2 \Lambda_5}{6} + \frac{C}{a_0^4} \quad (5)$$

and

$$\ddot{a}_0 = -\frac{\kappa^4 \rho_0^2}{12 a_0^7} + \frac{\kappa^2 \Lambda_5}{6} a_0 - \frac{C}{a_0^3} \quad (6)$$

Here C appears as an integration constant.

We now proceed to solve the equation (5). The integration leads to

$$\frac{1}{4} \int \frac{du}{\sqrt{bu^2 + Cu + d}} = \pm (t - t_0) \quad (7)$$

with $u = a_0^4$, $b = \frac{\kappa^2 \Lambda_5}{6}$, $d = \frac{\kappa^4 \rho_0^2}{36}$.

The explicit solutions for $b > 0$ are then given by

$$a_0^4 = \begin{cases} \frac{\sqrt{4bd - C^2}}{2b} \sinh \left[4\sqrt{b} (t - t_0) \right] - \frac{C}{2b}, & \text{(when } 4bd > C^2) \\ \frac{\sqrt{C^2 - 4bd}}{2b} \cosh \left[4\sqrt{b} (t - t_0) \right] - \frac{C}{2b}, & \text{(when } 4bd < C^2) \\ \frac{1}{2b} \left[e^{4\sqrt{b} (t - t_0)} \right] - \frac{C}{2b}, & \text{(when } 4bd = C^2) \end{cases} \quad (8)$$

We note that $b > 0$ implies positive bulk cosmological constant ($\Lambda_5 > 0$). Now for $C > 0$, all the above solutions start from big bang singularity and expand indefinitely as $t \rightarrow \infty$. However, for $C < 0$, the behaviour of the third solution is quite different. It is then a singularity free solution which starts with finite a_0 at $t \rightarrow -\infty$, where both \dot{a}_0 and \ddot{a}_0 vanish and subsequently shows exponential expansion. The brane model corresponding to this solution is termed as an emergent universe [1-3] in the brane world scenario.

We note that in the equation (5), the term including C is called the black radiation term, contributed from the bulk Weyl tensor. This parameter C arises from the analysis of the bulk [8, 10-12].

The five dimensional bulk space-time with flat spatial section may be written in the form (see Ida [13])

$$ds^2 = -h(a)dt^2 + \frac{1}{h(a)} da^2 + a^2 [d\chi^2 + \chi^2 d\Omega^2] \quad (9)$$

We may locate the three brane in the form of $t = t(\tau)$, $a = a(\tau)$ parametrized by the proper time τ on the brane, there the induced metric of three brane will be given by

$$ds_{(4)}^2 = -d\tau^2 + a^2(\tau) [d\chi^2 + \chi^2 d\Omega^2] \quad (10)$$

where τ and $a(\tau)$ correspond to the cosmic time and scale factor respectively for spatially flat Friedmann-Robertson-Walker universe.

The solution of the bulk field equations $R_{\mu\nu}^{(5)} = \Lambda_5 g_{\mu\nu}^{(5)}$ is shown clearly in [13] that for $\Lambda_5 > 0$ and $C < 0$, the bulk space-time becomes Schwarzschild-de Sitter type and the black hole horizon exists, where $h(a) = 0$. In fact, C plays the role of the black hole mass. It is interesting to note that because of the positive magnitude of the bulk cosmological constant ($\Lambda_5 > 0$), the naked singularity can be avoided even if C is chosen to be negative in the emergent universe model and the bulk background has a horizon for the bulk singularity. We further note from equation (6) that, with $\Lambda_5 > 0$ and $C < 0$, the emergent universe model evolves into an accelerated expansion at the late stage.

Moreover, for negative cosmological constant (i.e, $b < 0$), the solution can be written as

$$a_0^4 = \frac{C}{2|b|} + \frac{\sqrt{4|b|d + C^2}}{2|b|} \begin{matrix} Sin \\ or \\ Cos \end{matrix} [4\sqrt{|b|} (t - t_0)] \quad (11)$$

which has familiar behaviour and is not of much interest in the present context.

We shall now discuss the properties of that solution in equations (8) representing a model for emergent universe. Asymptotically in the past (i.e, at $t \rightarrow -\infty$) the scale factor a_0 has the constant value $(\frac{|C|}{2b})^{1/4}$ and using equation (4), the matter density has initially the constant value

$$\rho_{bi} = \left(\frac{6\Lambda_5}{\kappa^2} \right)^{1/2} \quad (12)$$

Further, it should be noted that equation (5) may also be written as

$$\frac{a_0^2}{a_0^2} = \frac{\kappa^4 \rho^2}{36} + \frac{\kappa^4 \sigma \rho}{18} + \frac{\kappa^2 \Lambda_4}{6} \quad (13)$$

where $\Lambda_4 = \Lambda_5 + \frac{\kappa^2 \sigma^2}{6}$, is the effective four dimensional cosmological constant and is non-zero positive constant due to positive Λ_5 .

In the present work, we have followed the procedure adopted by Binétruy et al. [7-8], in which the Randall-Sundrum spacetime was generalized to allow for time dependent cosmological expansion. We are mainly concerned with the derivation of the four dimensional brane cosmological equations using the Z_2 symmetry at the boundary. Using Gauss-Codazzi formulation with Israel's junction conditions [14] on the brane, Roy Maartens [15] and Shiromizu et al. [16] have formulated a pure brane Einstein equations, where the bulk geometry reflects through the projected Weyl tensor term. It should be noted that this approach yields exactly the same results as [7, 8] for FRW branes.

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